Question 1 [5 marks]

Let $\underset{\sim}{a} = 2\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$ and $\underset{\sim}{b} = -2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$.

a. Find a unit vector in the direction of $\underset{\sim}{a}$.

1 mark

Let Π be the plane passing through the origin, O, and containing the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

b. Find the cartesian equation of Π.

2 marks

c. Show that all vectors that are normal to Π are perpendicular to the vector $\underset{\sim}{c}$, where

$$\underset{\sim}{c} = m\underset{\sim}{a} + n\underset{\sim}{b}$$

and $m, n \in R\backslash\{0\}$.

2 marks

Question 1 [5 marks]

Let $\underset{\sim}{a} = 2\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$ and $\underset{\sim}{b} = -2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$.

a. Find a unit vector in the direction of $\underset{\sim}{a}$.

1 mark

Solution:

- Find the **magnitude** of $\underset{\sim}{a}$.

  $|\underset{\sim}{a}| = \sqrt{2^2 + 1^2 + (-2)^2}$

  $|\underset{\sim}{a}| = 3$

- Use the formula $\hat{\underset{\sim}{a}} = \frac{\underset{\sim}{a}}{|\underset{\sim}{a}|}$

  $\hat{\underset{\sim}{a}} = \frac{1}{3}\left(2\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}\right)$

Let Π be the plane passing through the origin, O, and containing the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

b. Find the cartesian equation of Π.

2 marks

Solution:

- Find a **normal vector** to the plane Π

  $\underset{\sim}{n} = \underset{\sim}{a} \times \underset{\sim}{b}$

  $\underset{\sim}{n} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{vmatrix}$

  $\underset{\sim}{n} = 3\underset{\sim}{i} + 6\underset{\sim}{j} + 6\underset{\sim}{k}$

- Use the formula $\mathbf{\underset{\sim}{0}} \cdot \mathbf{\underset{\sim}{n}} = \mathbf{\underset{\sim}{r}} \cdot \mathbf{\underset{\sim}{n}}$ where $\mathbf{\underset{\sim}{r}} = \mathbf{x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}}$

  $0 = 3x + 6y + 6z$

  $\therefore \Pi: 0 = x + 2y + 2z$

c. Show that all vectors that are normal to Π are perpendicular to the vector $\underset{\sim}{c}$, where

$$\underset{\sim}{c} = m\underset{\sim}{a} + n\underset{\sim}{b}$$

and $m, n \in R\backslash\{0\}$.

2 marks

Solution:

- Define **all vectors** that are **normal** to Π.

  $\underset{\sim}{n} = \lambda\left(3\underset{\sim}{i} + 6\underset{\sim}{j} + 6\underset{\sim}{k}\right) = \lambda(\underset{\sim}{a} \times \underset{\sim}{b})$ where $\lambda \in R\backslash\{0\}$

- Use the **dot product.**

  $\underset{\sim}{n} \cdot \underset{\sim}{c} = \lambda(\underset{\sim}{a} \times \underset{\sim}{b}) \cdot (m\underset{\sim}{a} + n\underset{\sim}{b})$.

  $\underset{\sim}{n} \cdot \underset{\sim}{c} = \lambda m(\underset{\sim}{a} \times \underset{\sim}{b}) \cdot \underset{\sim}{a} + \lambda n(\underset{\sim}{a} \times \underset{\sim}{b}) \cdot \underset{\sim}{b}$ using properties of the dot product.

  $\underset{\sim}{n} \cdot \underset{\sim}{c} = \lambda m \times 0 + \lambda n \times 0$ using properties of the cross product.

  $\underset{\sim}{n} \cdot \underset{\sim}{c} = 0$

  Therefore, $\underset{\sim}{n} \perp \underset{\sim}{c}$

Note: For *show that* questions, make sure you clearly define your parameters (such as $\lambda \in R\backslash\{0\}$ in the case above). Also – this is optional – try to specify any particular properties or theorems you use to **justify** each line of working.